Different Aspects of Analysis and Probability

DAAP 2016

September 26-27, 2016

Rzeszów, Poland

Book of Abstracts

UNIVERSITY OF RZESZÓW

FACULTY OF MATHEMATICS AND NATURAL SCIENCES

BOOK OF ABSTRACTS

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DIFFERENT ASPECTS OF ANALYSIS AND PROBABILITY
DAAP 2016

September 26-27, 2016, Rzeszów, Poland

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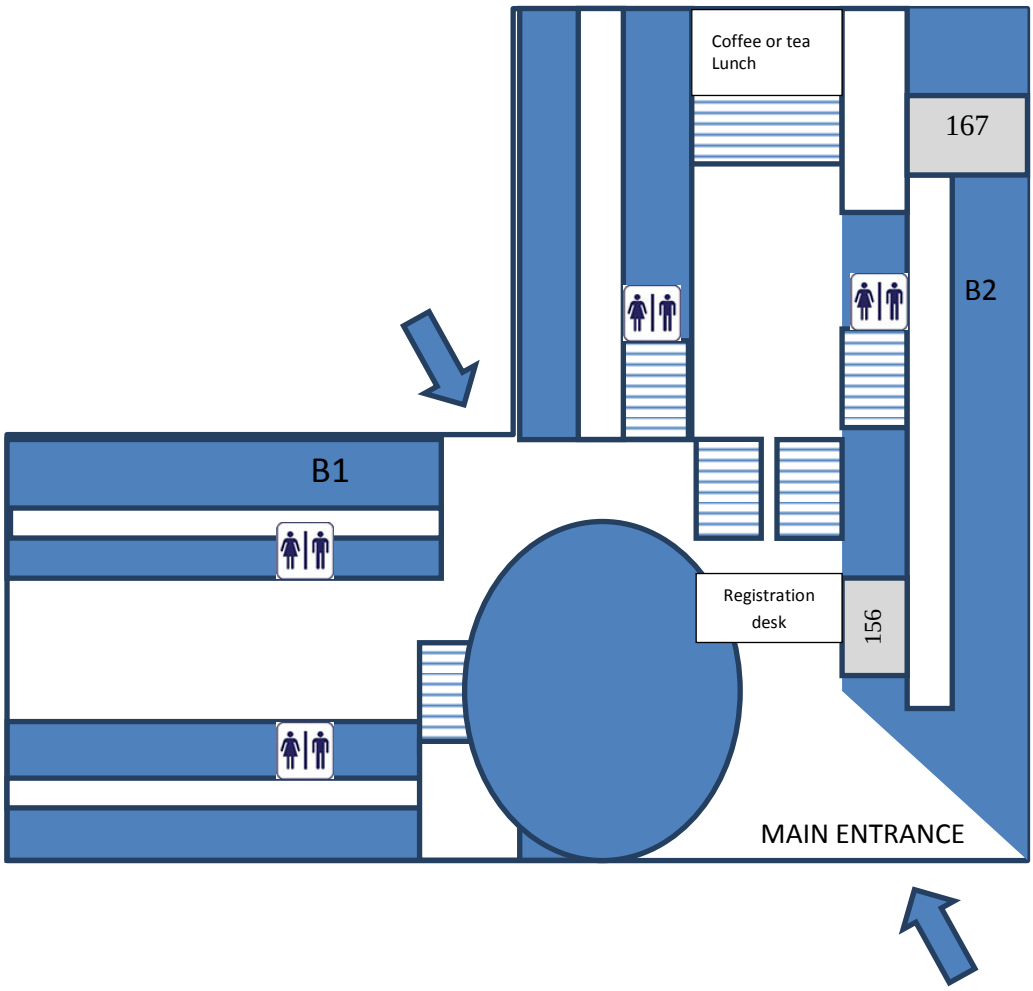
Programme of the Conference DAAP 2016

Sunday, September 25	
15:00 - 19:00	Registration
Monday, September 26	
8:00 - 8:55	Registration
Room 167 (B2, 1st floor)	
9:00 - 9:30	Opening
First morning session **Chairman**: Andrzej Kamiński	
9:30 - 10:10	Plenary lecture: Jasson Vindas *Recent developments on complex Tauberian theorems for Laplace transforms*
10:15 - 10:35	Contributed talk: Richard Carmichael *The finite Fourier transform for distributions and a conjecture concerning boundary values of vector valued analytic functions*
10:40 - 11:10	Coffee break
Second morning session **Chairman**: Wiesław Żelazko	
11:10 - 11:50	Plenary lecture: Stevan Pilipović *G-type spaces of ultradistributions over* $\mathbb{R}^d_+$
11:55 - 12:15	Contributed talk: Roman Frič *Probability: from Boolean to Łukasiewicz random events*
12:20 - 12:40	Contributed talk: Andrzej Łuczak *Ergodic projections in quantum dynamical semigroups*
13:00 - 14:00	Lunch break
Afternoon session **Chairman**: Marek Bożejko	
14:05 - 14:45	Plenary lecture: Władysław Wilczyński *Modes of convergence and local properties of measurable sets*
14:50 - 15:10	Contributed talk: Rostyslav Hryniv *On reflectionless Schrödinger operators*
15:15 - 15:35	Contributed talk: Wiesław Żelazko *A short history of Polish Mathematics*
Assembly hall (A1, ground floor)	
15:40 - 16:10	Coffee break
Anniversary of Prof. Andrzej Kamiński	
16:15 - 18:05	Special session
18:30 - 18:45	Transport for dinner
19:00	Conference dinner

Tuesday, September 27	
Room 167 (B2, 1st floor)	
First morning session **Chairman**: Richard Carmichael	
9:00 - 9:40	Plenary lecture: Marek Bożejko *Deformed Fock spaces, Hecke operators and monotone Fock space of Muraki*
9:45 - 10:05	Contributed talk: Adam Paszkiewicz *Paradoxes of divergence in Hilbert space, the magic touch of dilations*
10:10 - 10:30	Contributed talk: Mykhaylo Zarichnyi *Hyperspaces of convex sets and Eilenberg-MacLane spaces*
10:30 - 11:00	Coffee break
Second morning session **Chairman**: Władysław Wilczyński	
11:00 - 11:20	Contributed talk: Wiesław Żelazko *Concerning maximal abelian subalgebras of a Banach algebra*
11:25 - 11:45	Contributed talk: Svetlana Mincheva-Kaminska *Products of generalized functions in certain Gelfand-Shilov spaces*
11:50 - 12:10	Contributed talk: Soheyb Milles *The fixed point property for certain classes of intuitionistic fuzzy ordered sets*
12:15 - 12:35	Contributed talk: Hanna Podsędkowska *Properties and some application meaning of Segal's entropy for general quantum system*
Room 156 (B2, 1st floor)	
12:40 - 13:10	Poster session: Jacek Dziok, Katarzyna Halik, Monika Homa, Joanna Kowalczyk & Edyta Trybucka, Jacek Kucab, Ewa Rak, Sławomir Sorek, Dušana Štiberová, Rafał Wieczorek, Marek Żołdak
13:20 - 14:20	Lunch


Coffee or tea
Lunch
167
B2
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Registration
desk
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MAIN ENTRANCE

DEFORMED FOCK SPACES, HECKE OPERATORS AND MONOTONE FOCK SPACE OF MURAKI

MAREK BOŻEJKO

In this talk we will present the following subjects:

(1) Fock spaces of Yang-Baxter type.
   - Hecke operators and
   - positivity of T-symmetrizators.

(2) Connections of Woronowicz-Pusz operators T_μ^{CAR} with monotone Fock space of Muraki-Lu ($\mu = 0$).

(3) Non-commutative Levy process for generalized "ANYON" statistics.

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QUASI-ANALYTIC WAVE FRONTS

STEVAN PILIPOVIĆ

joint work with students and J. Toft

After recalling several definitions of wave - fronts, I will present the quasi-analytic wave-front set of distributions which correspond to the Gevrey sequence $p!^s$, $s \in [1/2, 1)$. Such wave-fronts are described through a sequences of Gaussian short-time Fourier transforms and suitable restriction-extension technique. Basic micro-local properties of the new wave-fronts are established.

The following estimate of the propagation of the wave-front, $s \in [1/2, 1)$, related to a distribution f and a differential operator with constant coefficients $P(D)$, is proved:

$$WF_s(P(D)f) \subseteq WF_s(f) \subseteq WF(s, P, f) \cup \mathrm{Char}(P),$$

where, $WF(s, P, f)$ is a suitable set determined by by the regularity of $P(D)(f^{rex})$ and the polynomial growth of the Fourier transform of f^{rex}.

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RECENT DEVELOPMENTS ON COMPLEX TAUBERIAN THEOREMS FOR LAPLACE TRANSFORMS

JASSON VINDAS

Complex Tauberian theorems for Laplace transforms have shown to be strikingly useful tools in diverse areas of mathematics such as number theory and spectral theory for differential operators. Many results in the area from the last three decades have been motivated by applications in operator theory and semigroups [1, 4].

In this lecture we shall discuss some recent developments on complex Tauberian theory for Laplace transforms and power series. We will focus on two groups of statements, usually labeled as Ingham-Fatou-Riesz theorems and Wiener-Ikehara theorems. Several classical applications will be discussed in order to explain the nature of these Tauberian theorems.

The results we will present considerably improve earlier Tauberians, on the one hand, by relaxing boundary requirements on Laplace transforms to local pseudofunction boundary behavior, with possible exceptional null sets of boundary singularities, and, on the other hand, by simultaneously considering one-sided Tauberian conditions. Using pseudofunctions allows us to take boundary hypotheses to a minimum, producing "if and only if" type results. In the case of power series, we will extend the Katznelson-Tzafriri theorem, one of the cornerstones in the modern asymptotic theory of operators [3].

The talk is based on collaborative work with G. Debruyne [2].

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MODES OF CONVERGENCE AND LOCAL PROPERTIES OF MEASURABLE SETS

WŁADYSŁAW WILCZYŃSKI

The definition of a density point of measurable set can be formulated in terms of the convergence in measure of a suitable sequence of characteristic functions. If instead of the convergence in measure another kind of the convergence is used (for example the convergence almost everywhere, the complete convergence, the pointwise convergence and so on) it is possible to obtain different kinds of density points as well as different topologies similar to the density topology.

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THE FINITE FOURIER TRANSFORM FOR DISTRIBUTIONS AND A CONJECTURE CONCERNING BOUNDARY VALUES OF VECTOR VALUED ANALYTIC FUNCTIONS

RICHARD D. CARMICHAEL

A finite Fourier transform of certain spaces of distributions is defined and properties of this transform are stated. Analytic functions in tubes in $\mathbb{C}^n$ are defined which are shown to obtain this finite Fourier transform as boundary value and properties of these analytic functions are proved. A conjecture concerning vector valued analytic functions which have boundary values in a space of tempered vector valued continuous linear transformations is considered; we conjecture that if the boundary value is a vector valued L^p function then the analytic function must be a vector valued Hardy H^p function.

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PROBABILITY: FROM BOOLEAN TO ŁUKASIEWICZ RANDOM EVENTS

ROMAN FRIČ

A categorical approach to probability theory leads to the extension of classical random events $\mathbf{A}$ - represented by a σ-field of sets - to generalized random events $\mathcal{M}(\mathbf{A})$ - represented by the measurable functions into [0,1] - and to the extension of Boolean logic to Łukasiewicz logic. We outline a model of generalized probability in which the classical probability space $(\Omega, \mathbf{A}, p)$ is replaced by $(\Omega, \mathcal{M}(\mathbf{A}), \int(.)\,\mathrm{d}p)$. The embedding of $\mathbf{A}$ into $\mathcal{M}(\mathbf{A})$ is, in some sense, analogous to the embedding of whole numbers into real numbers: it is a minimal divisible and complete extension.

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ON REFLECTIONLESS SCHRÖDINGER OPERATORS

ROSTYSLAV HRYNIV AND YAROSLAV MYKYTYUK

In his paper of 1949, V. Bargmann suggested several classes of isospectral Schrödinger operators on the line having the same negative bound states and absolutely continuous spectrum coinciding with the positive half-line. Several years later I. Gelfand, B. Levitan, V. Marchenko and M. Krein shaped up the inverse scattering theory, and it became clear that most of the Bargmann examples are given by reflectionless potentials. After C. S. Gardner, J. M. Greene, M. D. Kruskal and R. M. Miura discovered in 1967 the inverse scattering method of solving the famous Korteweg–de Vries (KdV) equation, the Bargmann potentials re-appeared as n-soliton solutions of KdV.

V. Marchenko (1991) and F. Gesztesy, W. Karwowski and Z. Zhao (1992) suggested several ways to generalize the notions of reflectionless Schrödinger operators and soliton solutions of the KdV equation. The aim of the talk is to report on our recent results in this direction. Namely, we suggest a method of constructing generalized reflectionless Schrödinger operators with arbitrary negative spectrum and soliton solutions of the corresponding KdV equation.

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ERGODIC PROJECTIONS IN QUANTUM DYNAMICAL SEMIGROUPS

ANDRZEJ ŁUCZAK

By a quantum dynamical semigroup is meant a semigroup $(T_t : t \in G)$ of unital positive normal linear maps on a von Neumann algebra. We assume that $G = (G, +)$ is an arbitrary abelian semigroup with unit, thus for the T_t's we have

$$T_{s+t} = T_s T_t, \quad s, t \in G.$$

The talk is aimed at presenting the role of ergodic projections in the investigation of ergodic properties of quantum dynamical semigroups. The novel feature is the construction of these projections with the help of general invariant means defined on the underlying semigroup G, as well as considering two kinds of the ergodic projections (called respectively "ergodic" and "quasi-ergodic") which together give a more thorough description of ergodic properties of quantum dynamical semigroups.

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THE FIXED POINT PROPERTY FOR CERTAIN CLASSES OF INTUITIONISTIC FUZZY ORDERED SETS

LEMNAOUAR ZEDAM, SOHEYB MILLES, AND EWA RAK

One of the important roles of the fixed point property is to combine properties of a set with the properties of maps in that set. In ordered sets theory, Tarski and Davis were the first who have studied the fixed point property for certain classes of ordered sets. In applications as point of view, the proof of fixed point results, whether for topological spaces and continuous functions or for ordered structures and order-preserving maps are very useful to show how one can work with a certain class of topological spaces or a certain ordered structure. For the new structures, the benefits are more, because the fixed point property can provide a familiar setting help us to investigate these new structures. Inspired by this, in this paper, we characterize the fixed point property for certain classes of intuitionistic fuzzy ordered sets previously introduced by Bustince and Burillo [1, 2]. Particularly, for the class of intuitionistic fuzzy lattices, recently proposed by Tripathy et al. [3], and for the class of intuitionistic fuzzy ordered chain-complete.

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PRODUCTS OF GENERALIZED FUNCTIONS IN CERTAIN GELFAND-SHILOV SPACES

SVETLANA MINCHEVA-KAMINSKA

We consider several sequential definitions of the product of distributions in the spaces $\mathcal{K}'(M_p)$ of Gelfand-Shilov as well as in the duals of the spaces of type $\mathcal{S}$ (see [4]). They are modifications of the Mikusiński-Shiraishi-Itano definitions of the product of distributions in $\mathcal{D}'$ (see [6, 7]) and are based on various classes of delta-sequences. Using the Mikusiński-Antosik diagonal theorem (see [1, 2]) we prove the equivalence of the considered definitions of the product in $\mathcal{K}'(M_p)$. This is a generalization of the result obtained in [5] for tempered distributions. Moreover, we prove that some analogous definitions of the product in the duals of the spaces $\Sigma_{N_q}^{M_p}$ and $\mathcal{S}_{N_q}^{M_p}$ are equivalent.

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PARADOXES OF DIVERGENCE IN HILBERT SPACE, THE MAGIC TOUCH OF DILATIONS

ADAM PASZKIEWICZ AND ANDRZEJ KOMISARSKI

Dedicated to Professor Andrzej Kamiński

The Amemiya-Ando conjecture on strong convergence of product of contraction $P_n \dots P_1$ for $n \to \infty$ and $P_1, P_2, \dots$ taken from a finite set $\{Q_1, \dots, Q_K\}$ of contractions in a Hilbert space was unsolved for 50 years. But the conjecture falls even for $\{Q_1, \dots, Q_k\}$ being a set of *three orthogonal projections*. The Naymark dilations theorem gives unexpectedly simple construction.

We shall present even more unexpected paradoxes for $\{Q_1, \dots, Q_k\}$ being conditional expectations, restricted to an L_2 space.

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PROPERTIES AND SOME APPLICATION MEANING OF SEGAL'S ENTROPY FOR GENERAL QUANTUM SYSTEM

HANNA PODSĘDKOWSKA AND RAFAŁ WIECZOREK

We consider Segal's definition of entropy of a normal state on a finite von Neumann algebra with faithful trace τ. Some important properties like invariance of entropy under trace preserving *-homomorphism, lower semi-continuity, additivity and subadditivity are presented. We also demonstrate application meaning of Segal's entropy in "Holevo-types" inequalities.

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ON THE APPROXIMATE SOLUTIONS OF THE FUZZY FRACTIONAL DIFFERENTIAL EQUATION

DJURDJICA TAKAČI AND ARPAD TAKAČI

The fuzzy fractional differential equations with fuzzy coefficients are analyzed in the frame of Mikusiński operators. The systems of fuzzy operational algebraic equation are obtained, in a view of the definition of fuzzy derivatives. Their exact and approximate solutions are constructed and their characters are analyzed considering them as the corresponding solutions of the given problem. The described procedure is illustrated on an example and the obtained approximate solutions of the considered problems are visualized by using GeoGebra software package.

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HYPERSPACES OF CONVEX SETS AND EILENBERG-MACLANE SPACES

MYKHAYLO ZARICHNYI

The Lie group S^1 acts by rotations on the hyperspace of convex bodies of constant width inscribed into the unit square as well as the hyperspaces of rotors in convex regular polygons.

The aim of the talk is to review some old and new results concerning the orbit spaces of these actions. It turns out that they contain some Q-manifolds that are Eilenberg-MacLane spaces.

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A SHORT HISTORY OF POLISH MATHEMATICS

WIESŁAW ŻELAZKO

In my talk I describe how Poland, having practically no mathematical traditions, got in a reasonably short time (1920-1939) a good international position in such fields as topology foundations of mathematics and functional analysis. In some detail I shall describe the Lwow School of functional analysis with Banach and Steihaus. If time permits, I shall describe also the looses during World War II and further rebuilding of mathematics in Poland.

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CONCERNING MAXIMAL ABELIAN SUBALGEBRAS OF A BANACH ALGEBRA

WIESŁAW ŻELAZKO

A commutative subalgebra $\mathcal{A}$ of a unital Banach algebra A is said maximal (shortly m.a.s) if there is no strictly larger abelian subalgebra of A. Such an m.a.s must contain the unity of A. Every commutative unital Banach algebra A is an m.a.s in $L(A)$ ([2]).

In my talk I give some results and formulate some questions concerning m.a.s. If time permits I shall indicate some proofs.

(1) Let X be a Banach space, $\dim X > 1$. Then each m.a.s in $L(X)$ has uncountably many copies, i.e. there are uncountably many m.a.s isomorphic with the given one ([5]).
(2) The above fails if instead of $L(X)$ we take an arbitrary non-commutative Banach algebra. There is an infinite dimensional Banach algebra A which has only one infinite dimensional m.a.s (such a subalgebra must exist by [3]), while all remaining ones are mutually isomorphic and of dimension two ([4]).
(3) There is an infinite dimensional unital Banach algebra in which all m.a.s are mutually isomorphic ([6]).
(4) A result obtained by Bračič and Kuzmič ([1]) states that no algebra $L(X)$, for an infinite dimensional Banach space X, can have a finitely dimensional m.a.s.
(5) There is a commutative unital Banach algebra $\mathcal{A}$ of dimension two, which can be an m.a.s only in two algebras, one of dimension three and another of dimension four ([7]).

Questions.

(i) When for two commutative Banach algebras $\mathcal{A}_1$ and $\mathcal{A}_2$ there is a Banach algebra A in which both are m.a.s ? The result in 5. shows that not all pairs are permitted.
(ii) For which pairs of commutative unital Banach algebras $\mathcal{A}_1, \mathcal{A}_2$ the fact that $\mathcal{A}_1$ is an m.a.s. in some Banach algebra A implies that $\mathcal{A}_2$ is also an m.a.s in A ?

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ON COMPLEX HARMONIC FUNCTIONS

JACEK DZIOK

Harmonic functions are famous for their use in the study of minimal surfaces and also play important roles in a variety of problems in applied mathematics. Recent interest in harmonic complex functions has been triggered by geometric function theorists Clunie and Sheil-Small [1].

A complex-valued continuous function $f : D \to \mathbb{C}$ is said to be harmonic in $D \subset \mathbb{C}$ if both functions $u := \operatorname{Re} f$ and $v := \operatorname{Im} f$ are real-valued harmonic functions in D. In any simply connected domain, we can write $f = h + \overline{g}$, where h and g are analytic in D.

The object of the present talk is to define and study classes of complex-valued functions which are harmonic in the unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. We give necessary and sufficient conditions, some topological properties and extreme points for these classes of functions. Moreover, by using extreme points theory we also obtain coefficients estimates, distortion theorems and integral mean inequalities.

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ON d-POSETS, d-σ-POSETS AND THEOREMS CONCERNING GENERATED SETS

KATARZYNA HALIK

We discuss the notions of d-posets and d-σ-posets which are natural extensions of D-posets and D-σ-posets, respectively, introduced by F. Kôpka and F. Chovanec, and prove certain theorems concerning various types of generated sets in d-σ-posets. In particular, we prove generalizations of several theorems of Sierpiński on various types of families generated by a given family of subsets of a set. The results are obtained in common with Aneta Dadej and Andrzej Kamiński.

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OSCILLATION THEORY FOR SINGULAR QUANTUM TREES

MONIKA HOMA AND ROSTYSLAV HRYNIV

We study the Sturm oscillation properties of quantum trees, i.e., of Sturm–Liouville operators $\tau := -\frac{d^2}{dx^2} + q$ defined on a metric tree Γ and subject to suitable matching and boundary conditions at the vertices of Γ. More specifically, q is a singular potential in $L^2(\Gamma)$, the matching conditions at the interior vertices are of Kirchhoff type, while the boundary conditions are of Robin type.

We show that the Sturm comparison theorem holds under minimal assumptions and the oscillation theorem holds generically, i.e., for operators with simple spectra.

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SECOND HANKEL DETERMINANT FOR CERTAIN SUBCLASS OF CLOSE-TO-CONVEX FUNCTIONS

JOANNA KOWALCZYK AND EDYTA TRYBUCKA

Let $\mathcal{K}_s(\gamma)$, $\gamma \in [0,1)$ denote the class of analytic functions f in the open unit disc $\mathcal{U}$ with the normalization $f(0) = f'(0) - 1 = 0$ and satisfying the condition

$$\mathfrak{Re}\left[\frac{-z^2 f'(z)}{g(z)g(-z)}\right] > \gamma, \quad z \in \mathcal{U},$$

for some g starlike of order $1/2$. For the class $\mathcal{K}_s(\gamma)$ the upper bound of the second Hankel determinant is found. Moreover the Fekete-Szegö problem is also studied.

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ON THE SUBPOWER COMPACTIFICATION OF A METRIC SPACE AND ITS PROPERTIES

JACEK KUCAB

We consider Higson subpower functions on a proper, unbounded metric space X, i.e., such continuous and bounded functions that diameters of images of balls vanish at infinity, even if the radius is controlled by the subpower function. These functions form a subalgebra in the algebra of all continuous and bounded functions. The compactification that corresponds to this subalgebra is called the (Higson) subpower compactification of X (see [4]). It is proved that this compactification is strictly greater than, researched by many authors (e.g. [1]), the (Higson) sublinear compactification. The remainder of the subpower compactification, so-called subpower Higson corona has many interesting properties. It is shown (see [3]), that with some additional conditions, the covering dimension of the subpower Higson corona equals the asymptotic power dimension of X (introduced in [2]). We also prove that there are no convergent sequences (except trivial ones) in this corona, so in particular, there is no embedding of a closed unit interval into it.

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SOME FUNCTIONAL EQUATIONS OF AGGREGATION OPERATIONS WITH ABSORBING ELEMENT

EWA RAK, LEMNAOUAR ZEDAM, AND SOHEYB MILLES

In this paper the functional equations of distributivity and modularity in a special class of aggregation operators with an absorbing element is investigated.
We focus first on the distributivity between binary operators from the family of 2-uninorms ($\mathbf{U}_{k(e,f)}$), which generalize nullnorms by extending their certain conditions. In particular, all possible solutions of the distributivity equation for the three defined subclasses of these operators depending on the position of its absorbing and neutral elements are characterized. Further on we examined the modularity equation for these operators. Depending on the position of its neutral elements and having the same absorbing element, we obtained both positive and negative results. They are analogous to the results in general case for operators with different absorbing elements.

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NEW RESULTS CONCERNING THE THEORY OF CIRCUITS AND THE OPERATIONS OF CONVOLUTION AND kTH POWER OF DISTRIBUTIONS

SŁAWOMIR SOREK

In the theory of linear systems and circuits, signal processing and telecommunications (see e.g. [3, 4]), the Dirac delta impulse appears to be very useful. This abstract signal $\delta = \delta(t)$, meant as a Schwartz distribution (see [5]), allows one to determine the input-output characteristics of non-autonomous linear systems and circuits as well as its impulse response.

In [1], a strict mathematical basis was presented for some aspects of the theory of linear and nonlinear systems and circuits with the domain of objects in use extended from functions to distributions to embrace δ, in particular. In case of nonlinear systems, expressed by infinite sums of homogeneous systems of degree k called the Taylor and Volterra series, the extension requires, in particular, a justification of the operations of the kth power and the convolution of kth order of the Dirac delta distribution.

This can be accomplished due to the notions of product and convolution of distributions in the sense of neutrix, the notion introduced by J. G. van der Corput (see [2]). In this presentation, new results concerning these operations are shown.

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TRIANGULAR NORMS AND CONORMS IN STATISTICS

DUŠANA ŠTIBEROVÁ

The main topic of the poster is an application of fuzzy mathematics in the statistical analysis of regression curve. The source of the poster is author's Diploma thesis, the goal of the thesis was to explore creative potential and applications of triangular norms and fuzzy numbers in the field of regression analysis. The poster provides an overview of important terms from the field of fuzzy mathematics, classical method of regression analysis ([1], [2], [3]) and theory of copulas. The core of the poster provides an overview of work of different authors in the field of fuzzy regression. Some of the described methods are illustrated on data from own statistical survey.

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MINIMIZING OF THE BAYES RISK IN THE QUANTUM CASE

RAFAŁ WIECZOREK

One of the important features of quantum mechanics is that nonorthogonal quantum states cannot be perfectly distinguished. Therefore, a fundamental problem in quantum mechanics is to design measurements optimized to distinguish between a collection of nonorthogonal quantum states. We consider the problem of finding a measurement which minimizes the Bayes risk for infinite (or finite) number of states on the von Neumann algebra. We show a solution of this problem for two states and some special cases. We present conditions for the existence, the uniqueness and the simplicity of an optimal measurement. We also demonstrate some estimation of the Bayes risk.

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OSTROWSKI'S AND MEHDI'S TYPE THEOREMS FOR APPROXIMATELY CONVEX FUNCTIONS

MAREK ŻOŁDAK

Let $(G,+)$ be an Abelian group, D be a subset of G and let $\alpha : D-D \to \mathbb{R}$. A function $f : D \to \mathbb{R}$ is called α-convex if

$$f(z) \leq \frac{f(x)+f(y)}{2} + \alpha(x-y)$$

for all $x, y, z \in D$ such that $x+y = 2z$.

A subset $D \subset G$ is called convex if for every $x, h \in G$ the condition $x+h, x-h \in D$ implies that $x \in D$.

We present the following two results concerning the connection between measurability and continuity of α-convex functions:

Theorem 1 [Ostrowski's type theorem]. Let G be an Abelian locally compact topological group, $D \subset G$ be an open, connected, convex set.
If $f : D \to \mathbb{R}$ is an α-convex function locally bounded above on a set $A \subset D$ of a positive Haar measure and α is locally bounded above at zero, then f is locally bounded. Moreover, if $\alpha(0) = 0$ and α is continuous at zero then f is locally uniformly continuous.

Theorem 2 [Mehdi's type theorem]. Let G be an Abelian topological group such that G is of the second category, $D \subset G$ be an open, connected, convex set.
If $f : D \to \mathbb{R}$ is an α-convex function locally bounded above on a set $A \subset D$ of the second category with the Baire property and α is locally bounded above at zero, then f is locally bounded. Moreover, if $\alpha(0) = 0$ and α is continuous at zero then f is locally uniformly continuous.

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